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Reg. No. : .....

Name : .....

Fourth Semester B.Tech. Degree Examination, May 2015
(2013 Scheme)
13.401 : ENGINEERING MATHEMATICS – III (BCHMNPSU)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** quesitons. **Each** question carries **4** marks.

1. Show that $w = \frac{1}{z}$ is analytic except at $z = 0$.

2. Evaluate $\int_C \frac{(1+z)\sin z}{(2z-1)^2} dz$ where $C : |z - i| = 2$ using Cauchy's integral formula.

3. Find the value of $\sqrt{35}$ using Newton-Raphson method.

4. Find the critical and fixed points of $w = \frac{5-4z}{4z-2}$.

5. Determine a, b, c, d so that function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.

PART – B

Answer **one full** question from each module. **Each** question carries **20** marks.

Module – I

6. a) Determine the analytic function whose real part is $u = e^{-x}(\cos y + \sin y)$. **7**

   b) Discuss about the conformal mapping $w = \sin z$. **7**

   c) Find the bilinear transformation which maps the points $z = 1, -i, i$ onto the points $2i, \infty, 0$. **6**

P.T.O.

7. a) State and prove the necessary conditions for a function $f(z) = u(x, y) + iv(x, y)$ to be analytic. 8

   b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. 5

   c) If f (z) is analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 2|f'(z)|^2$. 7

Module – II

8. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$ 10

   b) State Cauchy's residue theorem. Then calculate residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at $z = 1$. 10

9. a) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2}$. 10

   b) Find the Laurentz series $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $1 < |z| < 3$. 6

   c) Define the following :

      i) Poles

      ii) Essential singularity

      iii) Removable singularity. 4

Module – III

10. a) Solve the following system of equation by Gauss-Seidel method

      $8x + y + z = 8$, $2x + 4y + z = 4$, $x + 3y + 3z = 5$. 10

   b) Use Gauss forward interpolation formula to find y at $x = 0.68$ given that

x :	0.5	0.55	0.60	0.65	0.70	0.75	0.80
y :	0.1915	0.2088	0.2258	0.2422	0.2580	0.2734	0.2881

10

11. a) Find a positive root of $f(x) = x^3 + x^2 - 1$ by Bisection method correct to 4 places. 10

b) If $f(1) = 4$, $f(3) = 120$, $f(4) = 340$, $f(6) = 2544$ then find $f(5)$ using Lagrange Interpolation formula. 10

Module – IV

12. a) Solve the equation of $\frac{dy}{dx} = 1 - y$ given $y(0) = 0$ using Euler's modified method and Tabulate the solutions at $x = 0.1, 0.2$ and 0.3. 10

b) The velocity of a particle at distance 'S' from a point on its path is given by the table below.

S(in metre) :	0	10	20	30	40	50	60
V(m/sec) :	47	58	64	65	61	52	38

Estimate the time taken to travel 60 metres by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule. 10

13. a) Solve $\nabla^2 u = 0$ satisfying the following boundary conditions

$u(0, y) = 0$, $u(4, y) = 12 + y$ for $0 \le y \le 4$

$u(x, 0) = 3x$, $u(x, 4) = x^2$ for $0 \le x \le 4$. 10

b) Use Runge-Kutta 4^{th} order to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given $y(0) = 1$ at $x = 0.2, 0.4$. 10
